

Homomorphism Indistinguishability, Multiplicity Automata Equivalence, and Polynomial Identity Testing

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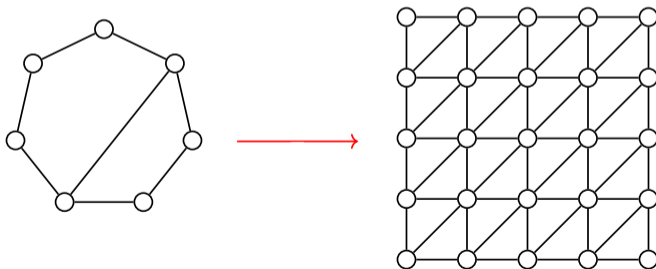
STACS, March 2026

Graph Homomorphisms

Definition

A (graph) homomorphism $F \rightarrow G$ is a function $\varphi : V(F) \rightarrow V(G)$ such that

$$uv \in E(F) \implies \varphi(u)\varphi(v) \in E(G).$$

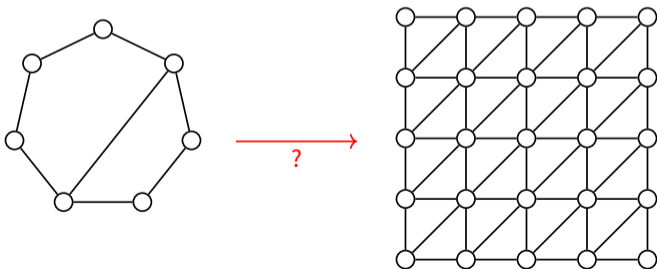


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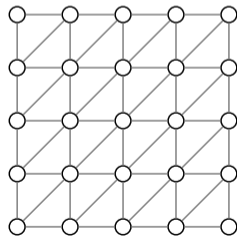
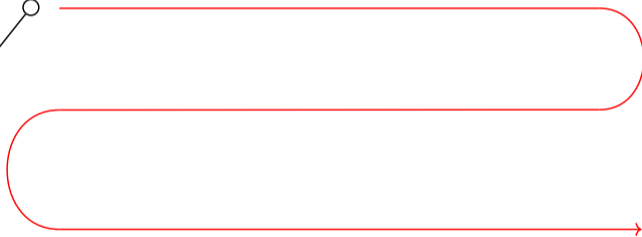
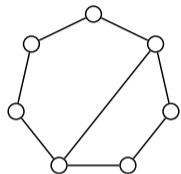
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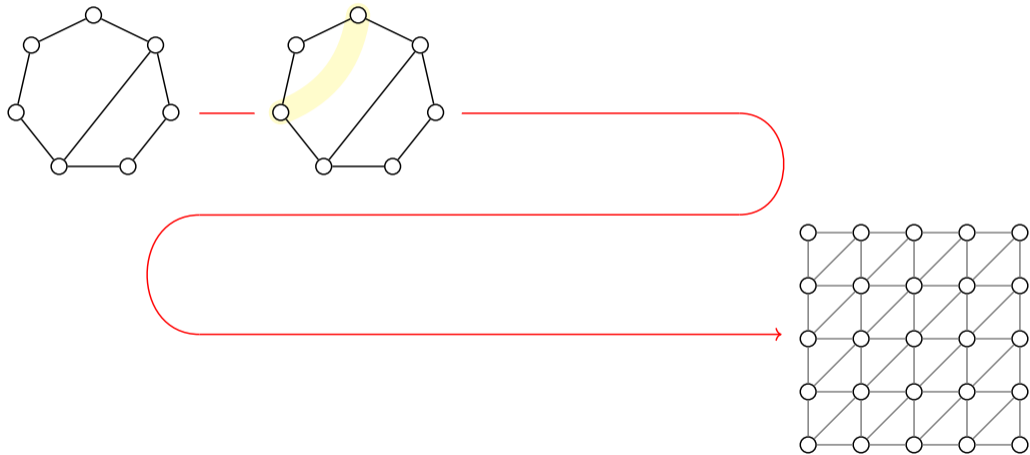
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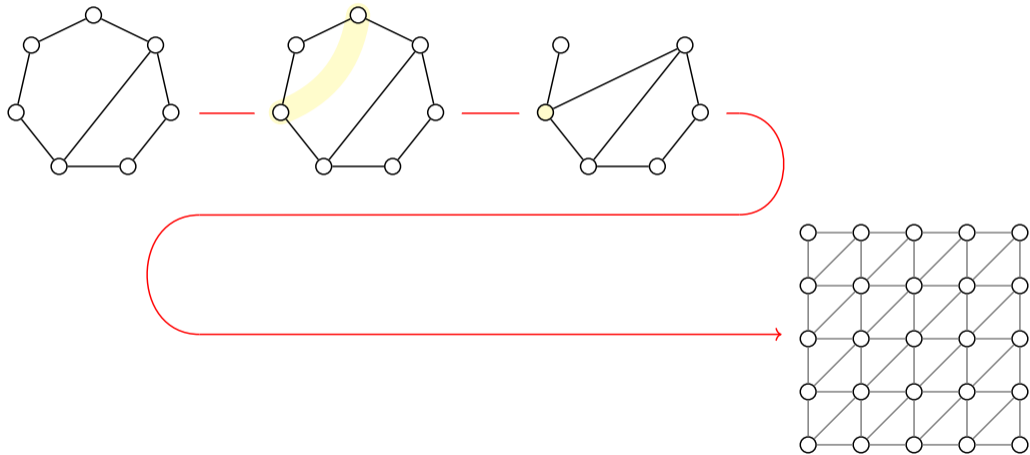
Example of a homomorphism $F \rightarrow G$.



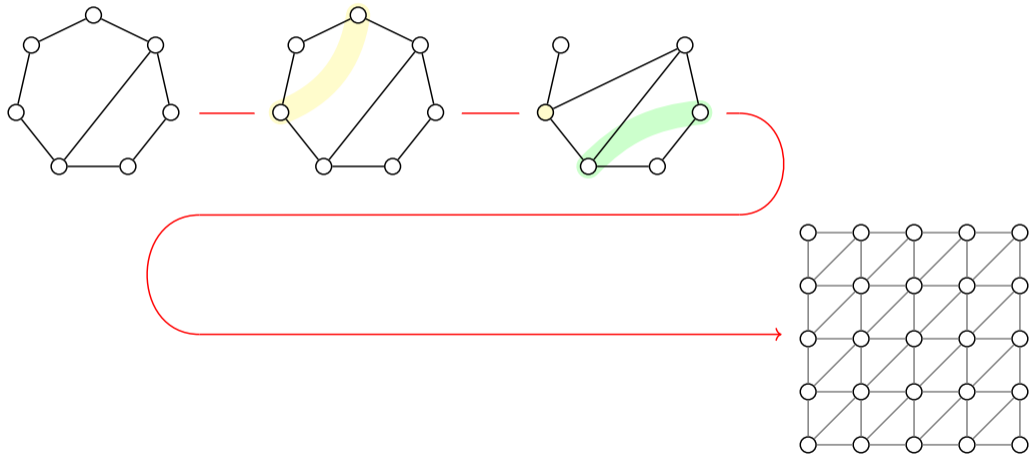
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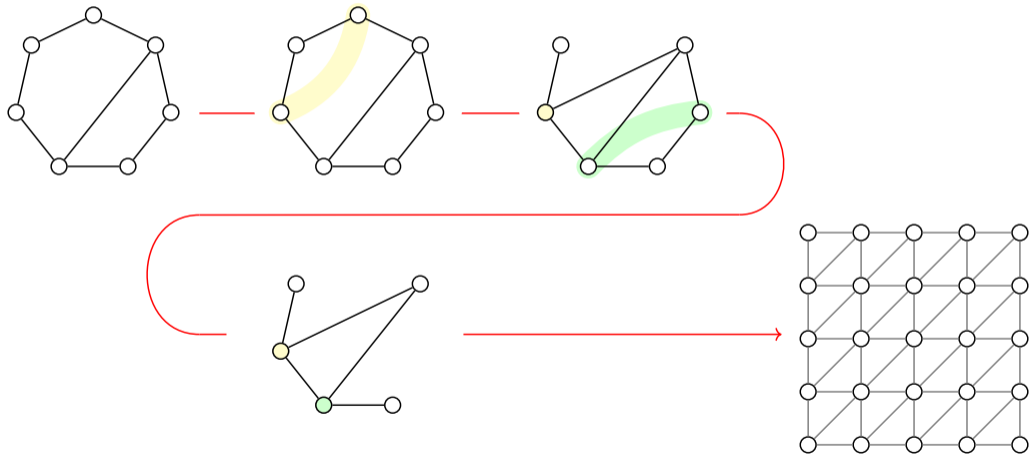
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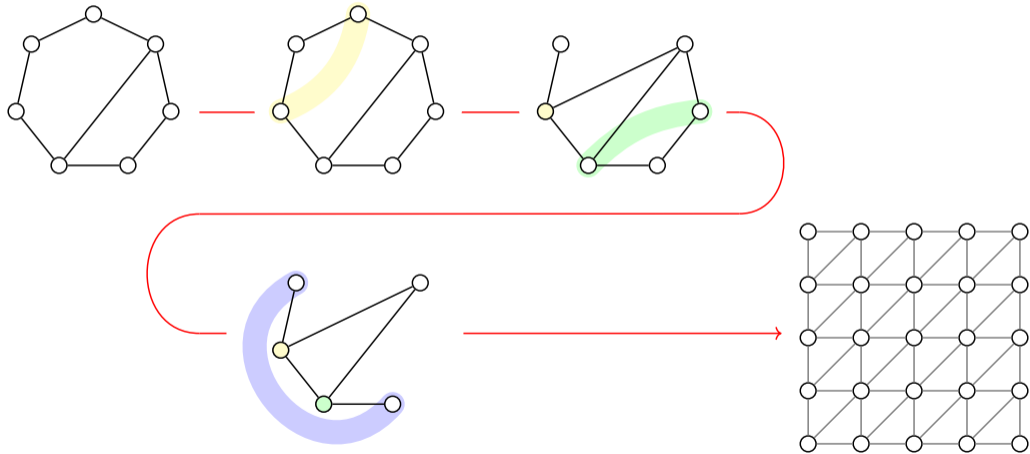
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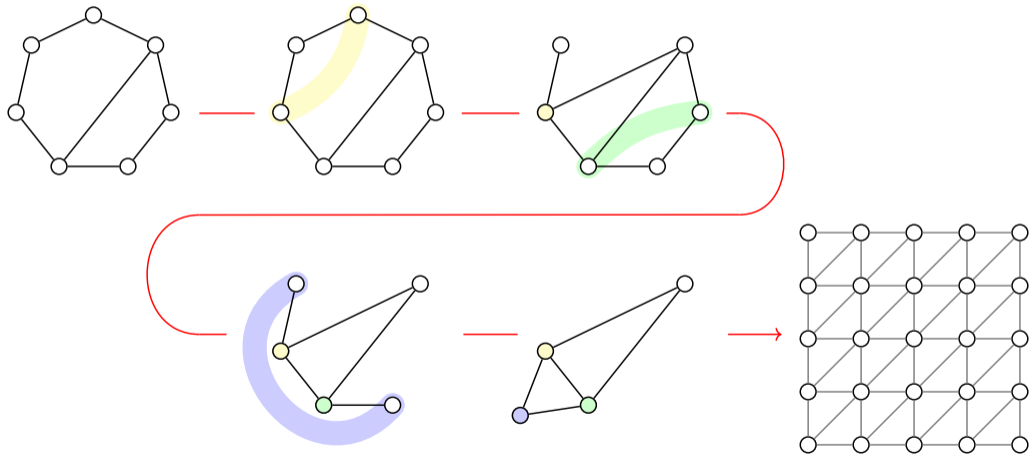
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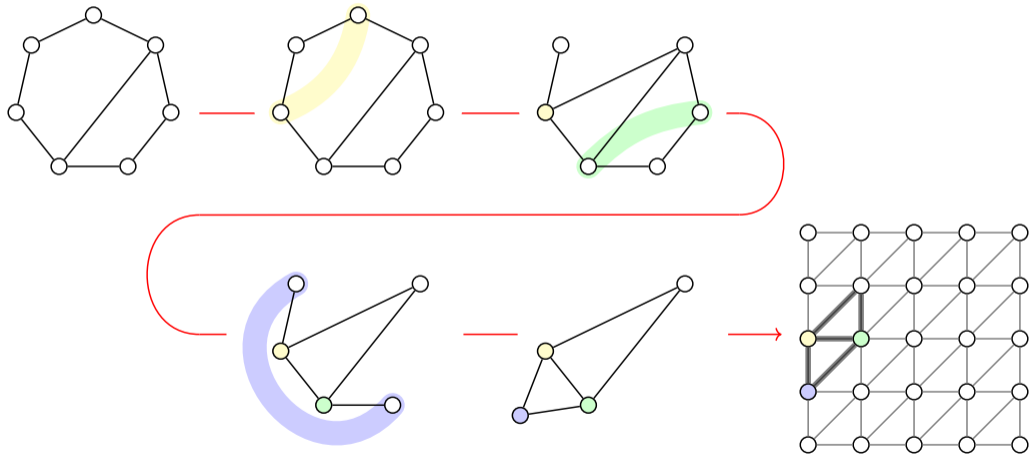
Example of a homomorphism $F \rightarrow G$.



Example of a homomorphism $F \rightarrow G$.



Example of a homomorphism $F \rightarrow G$.



Homomorphism Indistinguishability over \mathcal{F}

$\text{hom}(F, G)$... the number of homomorphisms $F \rightarrow G$.

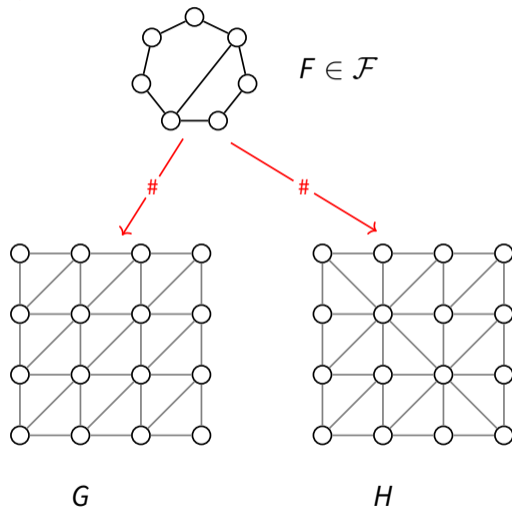
Definition ($G \equiv_{\mathcal{F}} H$)

$$\text{hom}(F, G) = \text{hom}(F, H) \quad \forall F \in \mathcal{F}.$$

Problem ($\text{HOMIND}(\mathcal{F})$)

Input: Graphs G and H .

Decide: $G \equiv_{\mathcal{F}} H$.



Connections of $\text{HOMIND}(\mathcal{F})$ to Other Problems

Graph class \mathcal{F} Problem $G \equiv_{\mathcal{F}} H$

See the HOMIND Zoo: tseppelt.github.io/homind-database.



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All graphs Graph Isomorphism

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<i>Treewidth $\leq k$</i>	k -dim Weisfeiler–Leman Test	Dvořák 2010
	$(k + 1)$ -variable FOL with counting	Cai, Fürer, and Immerman 1992

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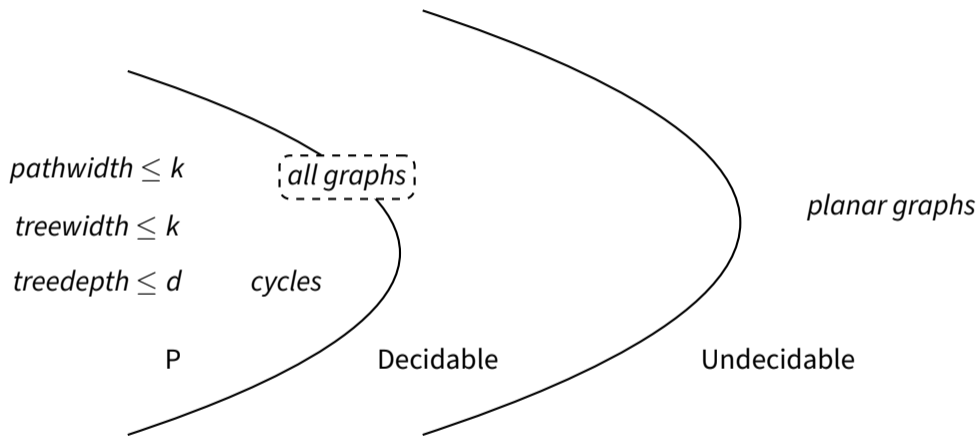
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<i>Cycles</i>	Cospectrality of adjacency matrices	folklore

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Complexity of $\text{HOMIND}(\mathcal{F})$



Results: Complexity under logspace many-one reductions (\implies)

$\text{HOMIND}(\mathcal{P})$
for recognisable \mathcal{P}
of bounded **pathwidth**

$\text{HOMIND}(\mathcal{T})$
for recognisable \mathcal{T}
of bounded **treewidth**

L NL C=L $L^{C=L}$ $L^{\#L}$ DET NC^2 NC P PIT coRP

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An Algorithmic Meta Theorem
for HOMIND , MFCS'24
Seppelt 2024

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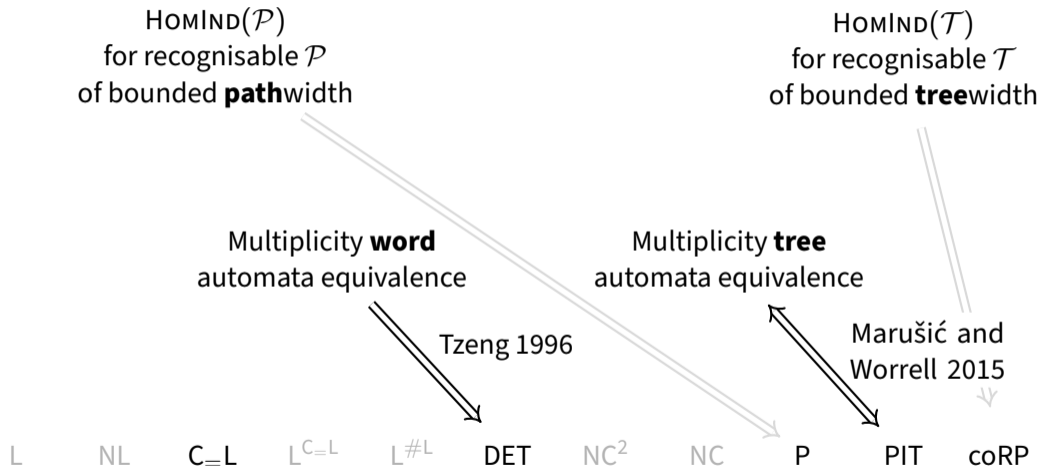
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Multiplicity **word**
automata equivalence

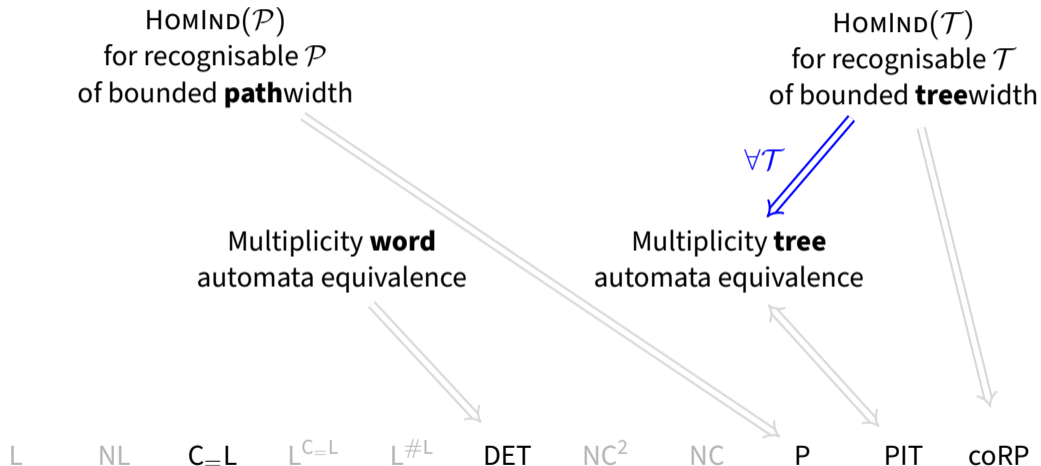
Multiplicity **tree**
automata equivalence

L NL C=L L^{C=L} L^{#L} DET NC² NC P PIT coRP

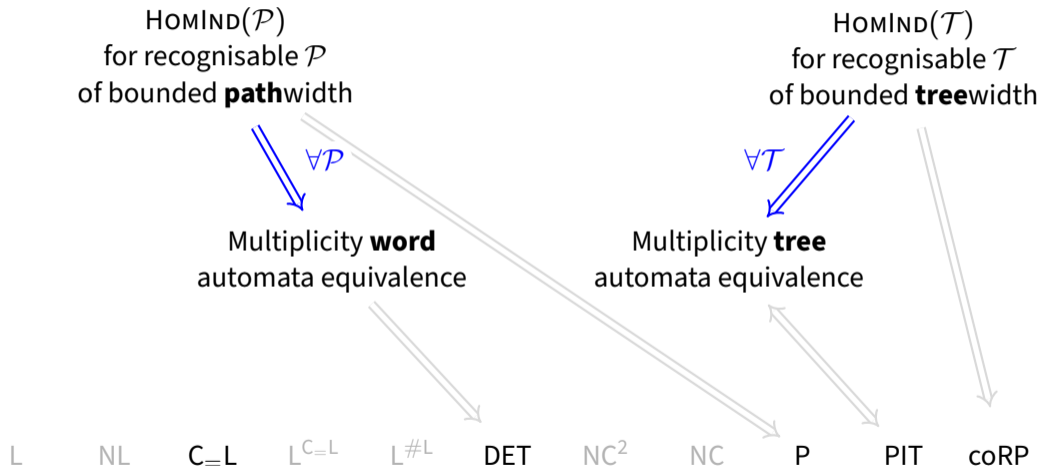
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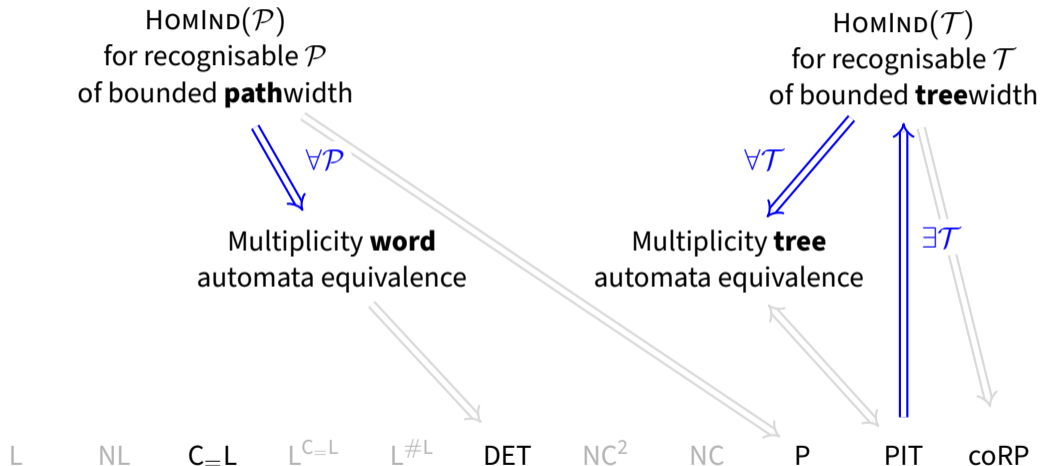
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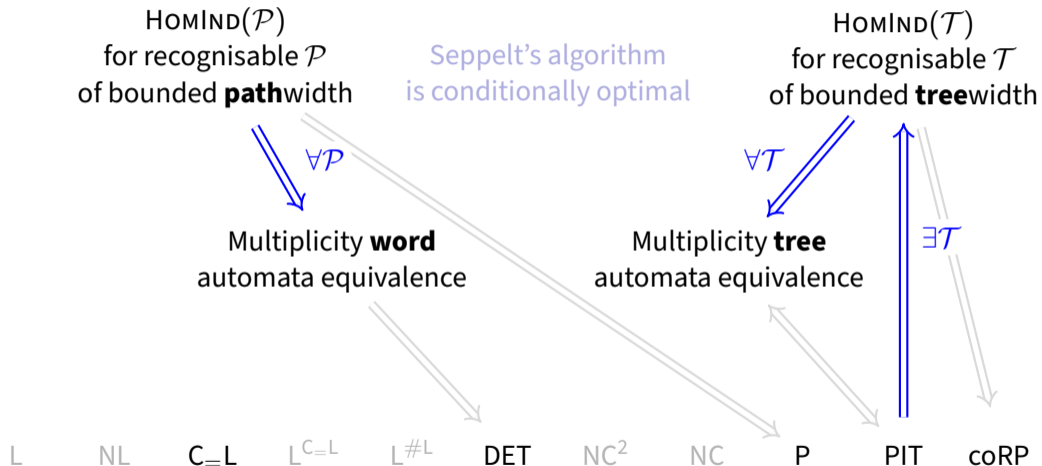
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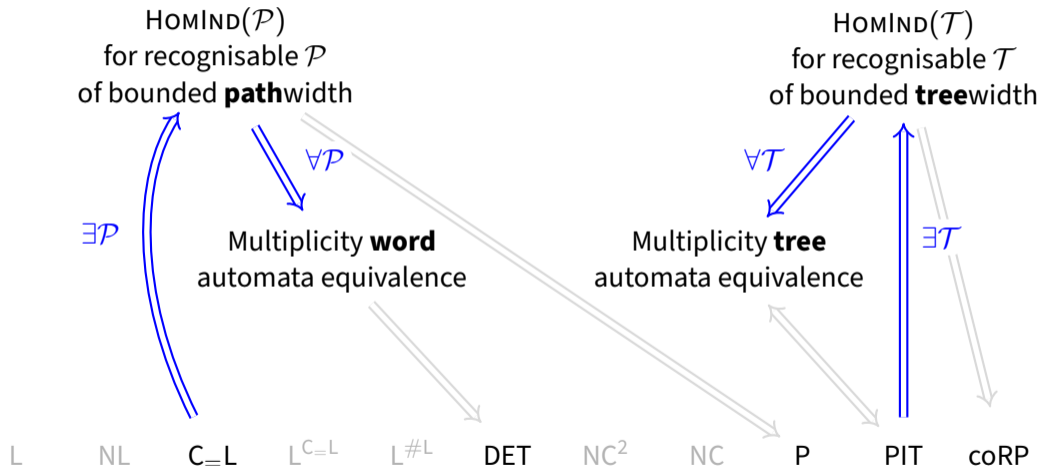
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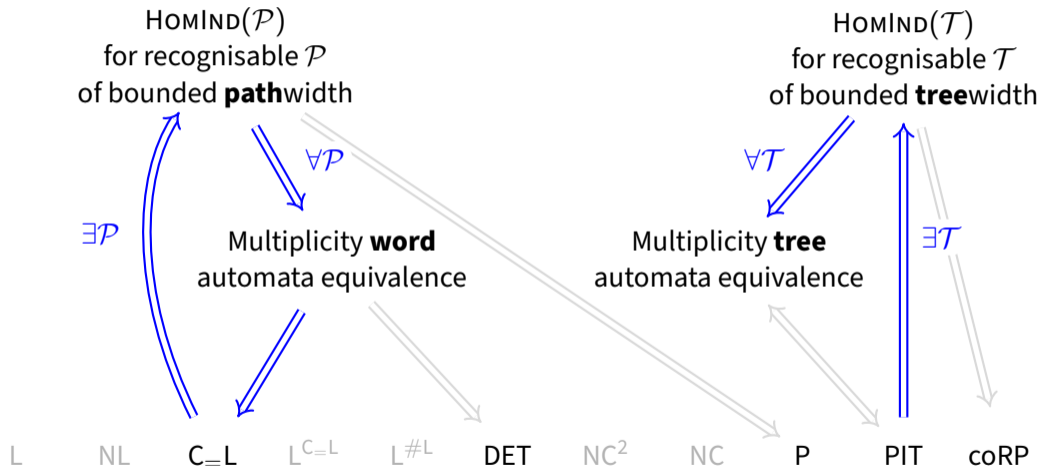
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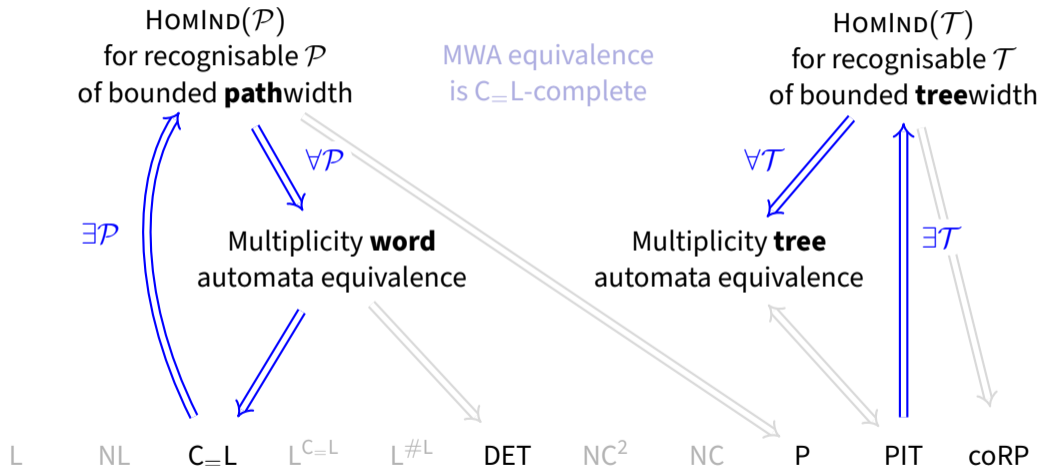
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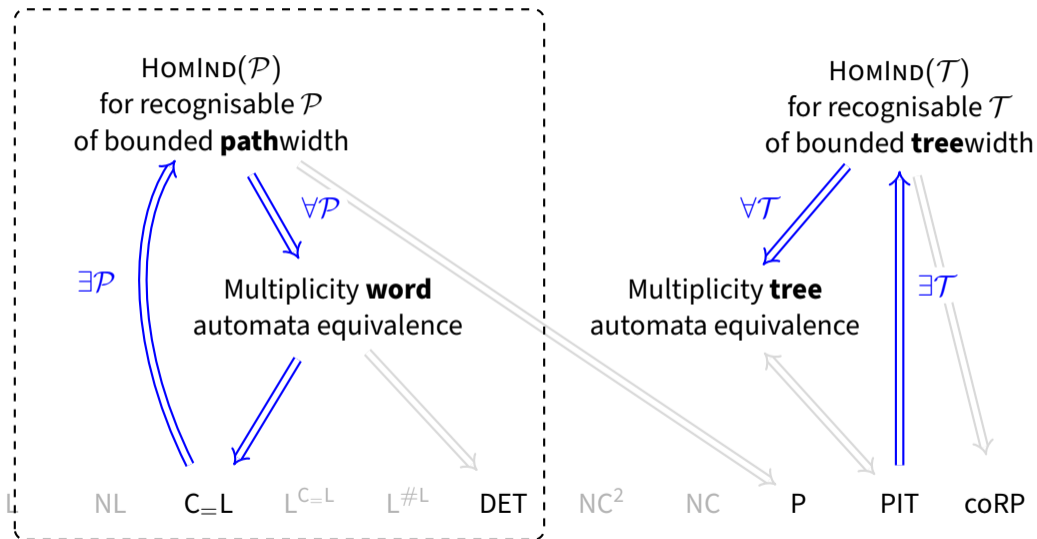
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Recognisability is CMSO₂-Definability

- Monadic Second-Order logic with modular counting predicates
- Examples of CMSO₂-definable graph classes:
 - treewidth $\leq k$, pathwidth $\leq k$, planar graphs, bounded degree, ...
 - minor-closed classes (using the Theorem of Robertson and Seymour 2004)

Definition (Recognisability, *informal*)

A property of a graph is *recognisable* if it can be recognised by a finite-state tree automaton.

Theorem

For every class of graphs \mathcal{F} of bounded treewidth holds

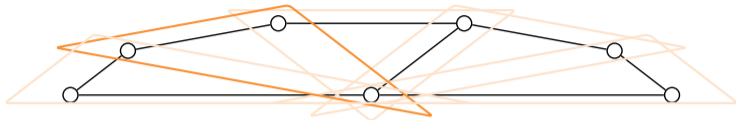
CMSO₂-definable \implies recognisable (Courcelle 1990),

CMSO₂-definable \longleftarrow recognisable (Bojańczyk and Pilipczuk 2016).

Recognising \mathcal{P} of Bounded Pathwidth



graph F

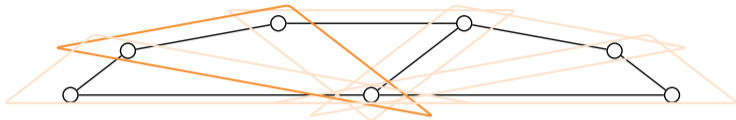


a path decomposition (P, β) of F of width $k = 3 - 1 = 2$

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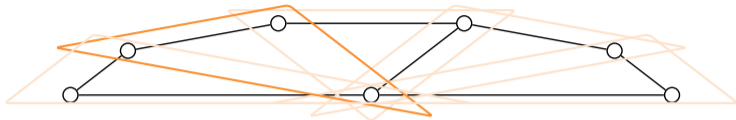
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- Alphabet of bags Σ . For $k = 2$, $\Sigma = \left\{ \begin{array}{c} \circ \\ \circ \\ \circ \end{array}, \begin{array}{c} \circ \\ \circ \\ \circ \end{array}, \dots, \begin{array}{c} \circ \\ \circ \\ \circ \end{array} \right\}$.

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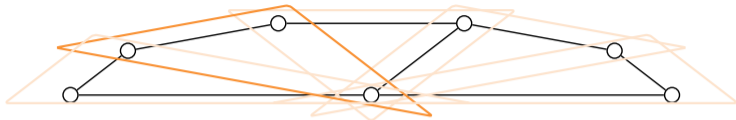
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$\mathcal{A}_{\mathcal{P}}$ (\mathcal{P} membership)

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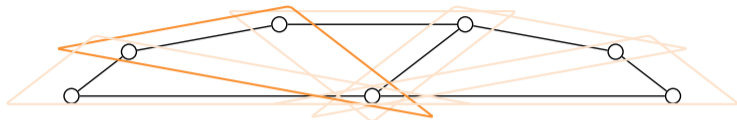
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\mathcal{A}_G (counting homomorphisms to G)

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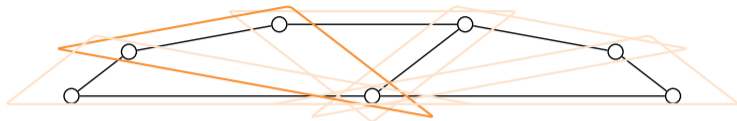
\mathcal{A}_G (counting homomorphisms to G)

- Recognises \mathcal{P} as a regular language $\Sigma^* \rightarrow \{0, 1\}$.
- $\llbracket \mathcal{A}_{\mathcal{P}} \rrbracket((P, \beta)) = \mathbf{1}_{\{F \in \mathcal{P}\}}$
- Bojańczyk and Pilipczuk 2016;
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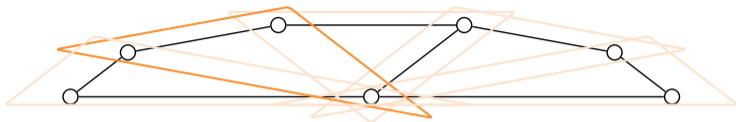
\mathcal{A}_G (counting homomorphisms to G)

- Recognises $\text{hom}(F, G)$ for each F of pathwidth $\leq k$ as a rational series over $\Sigma^* \rightarrow \mathbb{Q}$.
- $\llbracket \mathcal{A}_G \rrbracket((P, \beta)) = \text{hom}(F, G)$
- Seppelt 2024; Grohe, Rattan, and Seppelt 2025
(Homomorphism tensors)

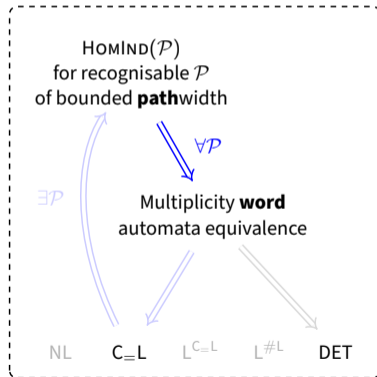
Recognising \mathcal{P} of Bounded Pathwidth (cont.)



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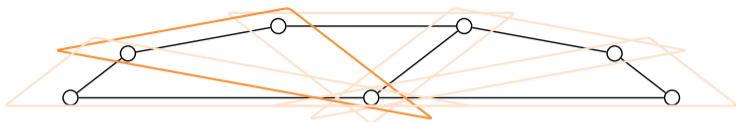
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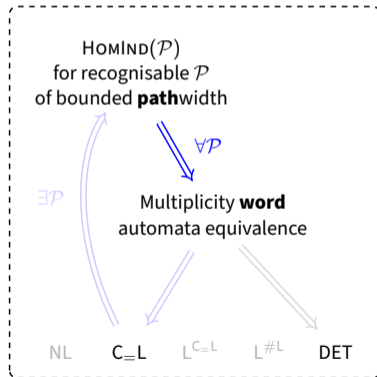


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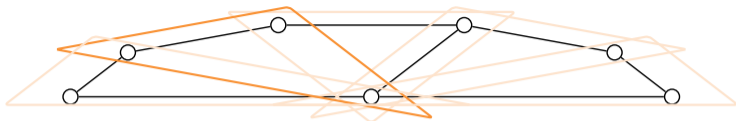
- Product $\mathcal{A}_{\mathcal{P}} \otimes \mathcal{A}_G$ of a DFA and an MWA.



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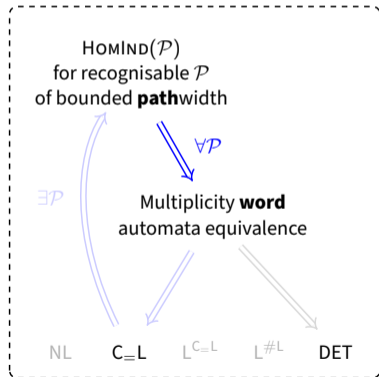


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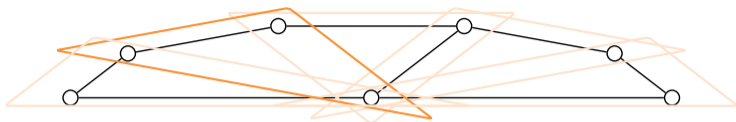
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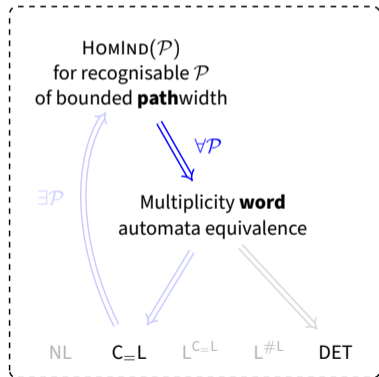


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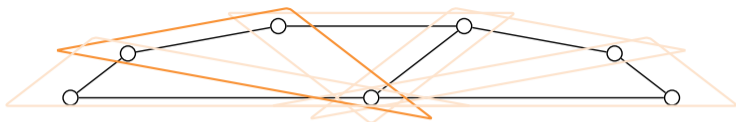
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- $\llbracket \mathcal{A}_{\mathcal{P}} \otimes \mathcal{A}_G \rrbracket : \Sigma^* \rightarrow \mathbb{Q}$.
- $\llbracket \mathcal{A}_{\mathcal{P}} \otimes \mathcal{A}_G \rrbracket ((P, \beta)) = \mathbf{1}_{\{F \in \mathcal{P}\}} \cdot \text{hom}(F, G)$
for each F with path decom. (P, β) of width $\leq k$.



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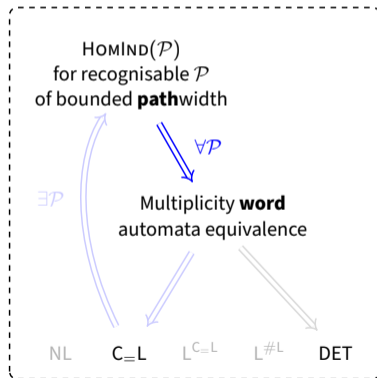
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- Product $\mathcal{A}_{\mathcal{P}} \otimes \mathcal{A}_G$ of a DFA and an MWA.
- $[[\mathcal{A}_{\mathcal{P}} \otimes \mathcal{A}_G]]: \Sigma^* \rightarrow \mathbb{Q}$.
- $[[\mathcal{A}_{\mathcal{P}} \otimes \mathcal{A}_G]]((P, \beta)) = \mathbf{1}_{\{F \in \mathcal{P}\}} \cdot \text{hom}(F, G)$
for each F with path decom. (P, β) of width $\leq k$.
- $\text{HOMIND}(\mathcal{P})$ as MWA equivalence (proof idea):

$$G \equiv_{\mathcal{P}} H \iff [[\mathcal{A}_{\mathcal{P}} \otimes \mathcal{A}_G]] = [[\mathcal{A}_{\mathcal{P}} \otimes \mathcal{A}_H]].$$



Multiplicity Word Automata (MWA)

Definition

Multiplicity word automaton is $\mathcal{A} = (S, \Sigma, M, \alpha, \eta)$

- states S , and alphabet Σ (both finite)
- $\alpha \in \mathbb{Q}^{1 \times S}$ (initial v.), $\eta \in \mathbb{Q}^S$ (final v.),
- $M: \Sigma \rightarrow \mathbb{Q}^{S \times S}$ (transition multiplicities).

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Definition (Semantics)

\mathcal{A} recognises the rational series $\llbracket \mathcal{A} \rrbracket: \Sigma^* \rightarrow \mathbb{Q}$

$$\llbracket \mathcal{A} \rrbracket(\sigma_1 \sigma_2 \cdots \sigma_t) = \alpha M(\sigma_1) M(\sigma_2) \cdots M(\sigma_t) \eta$$

for each word $\sigma_1 \sigma_2 \cdots \sigma_t \in \Sigma^*$.

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Problem (MWA Equivalence)

Input: MWAs \mathcal{A} and \mathcal{B} .

Decide: $\llbracket \mathcal{A} \rrbracket = \llbracket \mathcal{B} \rrbracket$.

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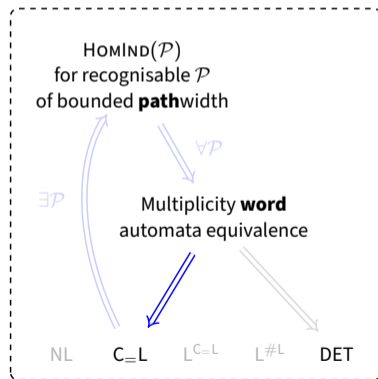
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What is $C=L$ class?

- Exact Counting Logspace $L \subseteq NL \subseteq C=L \subseteq L^{C=L} \subseteq L^{\#L} \subseteq DET \subseteq NC^2 \subseteq NC \subseteq P.$

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Input: Matrix $A \in \mathbb{Z}^{n \times n}, n \in \mathbb{N}.$

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- $L^{C=L}$ -complete problem
 - **Input:** Matrix $A \in \mathbb{Z}^{m \times n}$, vector $b \in \mathbb{Z}^m$, $n, m \in \mathbb{N}$.
Decide: $\exists x : Ax = b$.

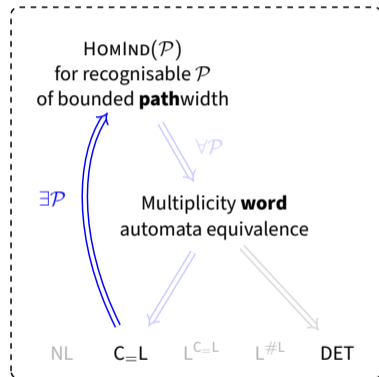
C=L-completeness of $\text{HOMIND}(\mathcal{P})$ for recognisable \mathcal{P} of bounded pathwidth (Proof idea)

Problem (Cospectrality of 0-1 matrices)

Input: matrices $A, B \in \{0, 1\}^{n \times n}$,

Decide: $\chi_A = \chi_B$.

- We show C=L-completeness of the above problem



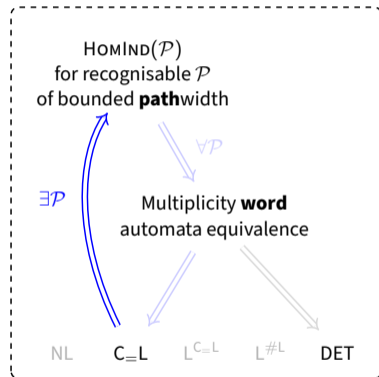
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- We show C=L-completeness of the above problem
 - reduce to non-negative matrices



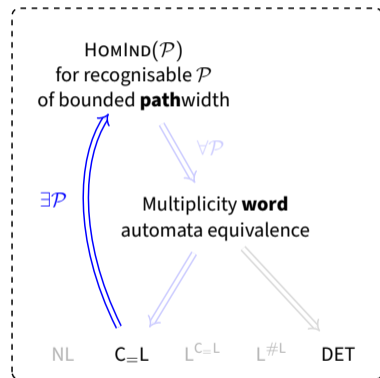
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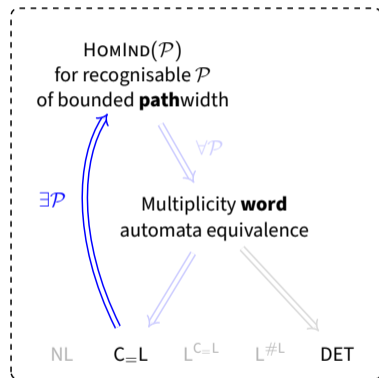
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C=L-completeness of $\text{HOMIND}(\mathcal{P})$ for recognisable \mathcal{P} of bounded pathwidth

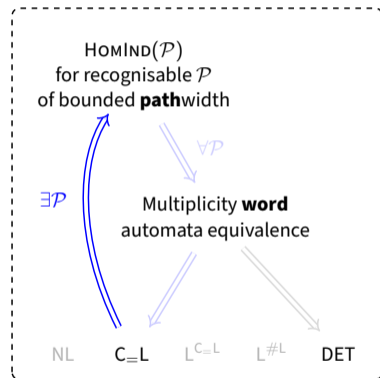
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- Newton's identities for $\sum_{i=1}^n \lambda_i^k = \text{tr}(A^k)$ imply
 - $\chi_A = \chi_B$ iff. $\text{tr}(A^k) = \text{tr}(B^k) \quad \forall k \in \mathbb{N}$.



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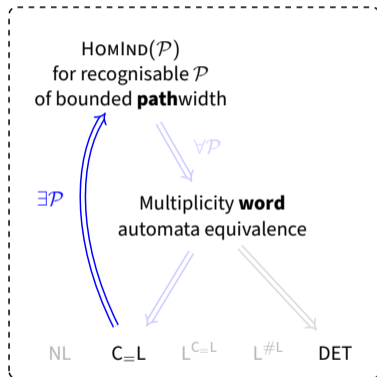
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- For the class of directed cycles \vec{C} ,



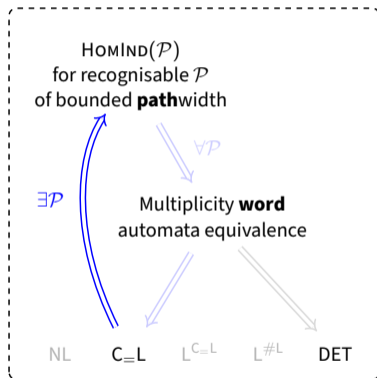
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Problem (Cospectrality of 0-1 matrices)

Input: matrices $A, B \in \{0, 1\}^{n \times n}$,

Decide: $\chi_A = \chi_B$.

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- Newtons identities for $\sum_{i=1}^n \lambda_i^k = \text{tr}(A^k)$ imply
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- For the class of directed cycles \vec{C} ,
 - $\text{hom}(\vec{C}_k, \vec{G}) = \text{tr}(A^k) \quad \forall \vec{C}_k \in \vec{C}$.



C=L-completeness of $\text{HOMIND}(\mathcal{P})$ for recognisable \mathcal{P} of bounded pathwidth

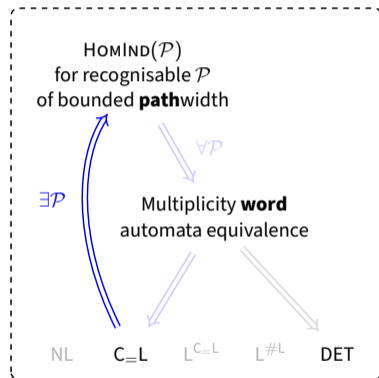
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Problem (Cospectrality of 0-1 matrices)





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- We show C=L-completeness of the above problem
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- Newton's identities for $\sum_{i=1}^n \lambda_i^k = \text{tr}(A^k)$ imply
 - $\chi_A = \chi_B$ iff. $\text{tr}(A^k) = \text{tr}(B^k) \quad \forall k \in \mathbb{N}$.
- For the class of directed cycles $\vec{\mathcal{C}}$,
 - $\text{hom}(\vec{\mathcal{C}}_k, \vec{G}) = \text{tr}(A^k) \quad \forall \vec{\mathcal{C}}_k \in \vec{\mathcal{C}}$.
 - Directed variant $\vec{G} \equiv_{\vec{\mathcal{C}}} \vec{H}$.







References I

-  Bojańczyk, Mikołaj and Michał Pilipczuk (July 2016). “Definability equals recognizability for graphs of bounded treewidth”. en. In: *Proceedings of the 31st Annual ACM/IEEE Symposium on Logic in Computer Science*. New York NY USA: ACM, pp. 407–416. ISBN: 978-1-4503-4391-6. DOI: 10.1145/2933575.2934508. (Visited on 03/16/2023).
-  Cai, Jin-Yi, Martin Fürer, and Neil Immerman (1992). “An optimal lower bound on the number of variables for graph identification”. In: *Combinatorica* 12.4, pp. 389–410. ISSN: 1439-6912. DOI: 10.1007/BF01305232.
-  Courcelle, Bruno (Mar. 1990). “The monadic second-order logic of graphs. I. Recognizable sets of finite graphs”. In: *Information and Computation* 85.1, pp. 12–75. ISSN: 0890-5401. DOI: 10.1016/0890-5401(90)90043-H.
-  Dvořák, Zdeněk (Aug. 2010). “On recognizing graphs by numbers of homomorphisms”. en. In: *Journal of Graph Theory* 64.4, pp. 330–342. ISSN: 03649024. DOI: 10.1002/jgt.20461. (Visited on 11/27/2020).

References II

-  Grohe, Martin (2020). “Counting Bounded Tree Depth Homomorphisms”. In: *Proceedings of the 35th Annual ACM/IEEE Symposium on Logic in Computer Science*. LICS ’20. New York, NY, USA: Association for Computing Machinery, pp. 507–520. ISBN: 978-1-4503-7104-9. DOI: [10.1145/3373718.3394739](https://doi.org/10.1145/3373718.3394739).
-  Grohe, Martin, Gaurav Rattan, and Tim Seppelt (Apr. 2025). “Homomorphism Tensors and Linear Equations”. en. In: *Advances in Combinatorics*. ISSN: 25175599. DOI: [10.19086/aic.2025.4](https://doi.org/10.19086/aic.2025.4). (Visited on 04/28/2025).
-  Lovász, László (Sept. 1967). “Operations with structures”. In: *Acta Mathematica Academiae Scientiarum Hungarica* 18.3, pp. 321–328. ISSN: 1588-2632. DOI: [10.1007/BF02280291](https://doi.org/10.1007/BF02280291).
-  Mančinska, Laura and David E. Roberson (2020). “Quantum isomorphism is equivalent to equality of homomorphism counts from planar graphs”. In: *IEEE 61st Annual Symposium on Foundations of Computer Science (FOCS)*, pp. 661–672. DOI: [10.1109/FOCS46700.2020.00067](https://doi.org/10.1109/FOCS46700.2020.00067).

References III

-  Marušić, Ines and James Worrell (2015). “Complexity of equivalence and learning for multiplicity tree automata”. In: *J. Mach. Learn. Res.* 16, pp. 2465–2500. DOI: [10.5555/2789272.2912078](https://doi.org/10.5555/2789272.2912078).
-  Robertson, Neil and P.D. Seymour (Nov. 2004). “Graph Minors. XX. Wagner’s conjecture”. In: *Special Issue Dedicated to Professor W.T. Tutte* 92.2, pp. 325–357. ISSN: 0095-8956. DOI: [10.1016/j.jctb.2004.08.001](https://doi.org/10.1016/j.jctb.2004.08.001).
-  Seppelt, Tim (2024). “An Algorithmic Meta Theorem for Homomorphism Indistinguishability”. In: *49th International Symposium on Mathematical Foundations of Computer Science (MFCS 2024)*. Ed. by Rastislav Královič and Antonín Kučera. Vol. 306. Leibniz International Proceedings in Informatics (LIPIcs). Dagstuhl, Germany: Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 82:1–82:19. ISBN: 978-3-95977-335-5. DOI: [10.4230/LIPIcs.MFCS.2024.82](https://doi.org/10.4230/LIPIcs.MFCS.2024.82).
-  Tzeng, Wen-Guey (Apr. 1996). “On path equivalence of nondeterministic finite automata”. In: *Information Processing Letters* 58.1, pp. 43–46. ISSN: 00200190. DOI: [10.1016/0020-0190\(96\)00039-7](https://doi.org/10.1016/0020-0190(96)00039-7). (Visited on 05/26/2025).